

# 积分公式大全

本附件所有公式来自《高等数学 (第三版)》

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## 1 含有 $ax + b$ 的积分

$$1. \int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b| + C$$

$$2. \int (ax+b)^\mu dx = \frac{1}{a(\mu+1)} (ax+b)^{\mu+1} + C \quad (\mu \neq -1)$$

$$3. \int \frac{x}{ax+b} dx = \frac{1}{a^2} (ax+b - b \ln |(x+b)|) + C$$

$$4. \int \frac{x^2}{ax+b} dx = \frac{1}{a^3} \left[ \frac{1}{2} (ax+b)^2 - 2b(ax+b) + b^2 \ln |ax+b| \right] + C$$

$$5. \int \frac{dx}{x(ax+b)} = -\frac{1}{b} \ln \left| \frac{ax+b}{x} \right| + C$$

$$6. \int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax+b}{x} \right| + C$$

$$7. \int \frac{x}{(ax+b)^2} dx = \frac{1}{a^2} \left( \ln |ax+b| + \frac{b}{ax+b} \right) + C$$

$$8. \int \frac{x^2}{(ax+b)^2} dx = \frac{1}{a^3} \left( ax+b - 2b \ln |ax+b| - \frac{b^2}{ax+b} \right) + C$$

$$9. \int \frac{dx}{x(a+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2} \ln \left| \frac{ax+b}{x} \right| + C$$

## 2 含有 $\sqrt{ax+b}$ 的积分

$$10. \int \sqrt{ax+b} dx = \frac{2}{3a} \sqrt{(ax+b)^3} + C$$

$$11. \int x\sqrt{ax+b}dx = \frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C$$

$$12. \int x^2\sqrt{ax+b}dx = \frac{2}{105a^3}(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3} + C$$

$$13. \int \frac{x}{\sqrt{ax+b}}dx = \frac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C$$

$$14. \int \frac{x^2}{\sqrt{ax+b}}dx = \frac{2}{15a^2}(2a^2x^2-4abx+8b^2)\sqrt{ax+b} + C$$

$$15. \int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \right| + C & (b > 0) \\ \frac{2}{-\sqrt{-b}} \arctan \sqrt{\frac{ax+b}{-b}} + C & (b < 0) \end{cases}$$

$$16. \int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}}$$

$$17. \int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}$$

$$18. \int \frac{\sqrt{ax+b}}{x^2}dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}}$$

### 3 含有 $x^2 \pm a$ 的积分

$$19. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$20. \int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2} \int \frac{dx}{(x^2+a^2)^{n-1}}$$

$$21. \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

4 含有  $ax^2 + b$  的积分

$$22. \int \frac{dx}{ax^2 + b} = \begin{cases} \frac{1}{\sqrt{ab}} \arctan \sqrt{\frac{a}{b}} x + C & (b > 0) \\ \frac{1}{2\sqrt{-ab}} \ln \left| \frac{\sqrt{ax} - \sqrt{-b}}{\sqrt{ax} + \sqrt{-b}} \right| + C & (b < 0) \end{cases}$$

$$23. \int \frac{x}{ax^2 + b} dx = \frac{1}{2a} \ln |ax^2 + b| + C$$

$$24. \int \frac{x^2}{ax^2 + b} dx = \frac{x}{a} - \frac{b}{a} \int \frac{dx}{ax^2 + b}$$

$$25. \int \frac{dx}{x(ax^2 + b)} = \frac{1}{2b} \ln \frac{x^2}{|ax^2 + b|} + C$$

$$26. \int \frac{dx}{x^2(ax^2 + b)} = -\frac{1}{bx} - \frac{a}{b} \int \frac{dx}{ax^2 + b}$$

$$27. \int \frac{dx}{x^3(ax^2 + b)} = \frac{a}{2b^2} \ln \frac{|ax^2 + b|}{x^2} - \frac{1}{2bx^2} + C$$

$$28. \int \frac{dx}{(ax^2 + b)^2} = \frac{x}{2b(ax^2 + b)} + \frac{1}{2b} \int \frac{dx}{ax^2 + b}$$

5 含有  $ax^2 + bx + c$  的积分

$$29. \int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} + C & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| + C & (b^2 > 4ac) \end{cases}$$

$$30. \int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c}$$

6 含有  $\sqrt{x^2 + a^2}$  ( $a > 0$ ) 的积分

$$31. \int \frac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arsh} \frac{x}{a} + C = \ln(x + \sqrt{x^2 + a^2}) + C$$

$$32. \int \frac{dx}{\sqrt{(x^2 + a^2)^3}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} + C$$

$$33. \int \frac{x}{\sqrt{x^2 + a^2}} dx = \sqrt{x^2 + a^2} + C$$

$$34. \int \frac{x}{\sqrt{(x^2 + a^2)^3}} dx = -\frac{1}{\sqrt{x^2 + a^2}} + C$$

$$35. \int \frac{x^2}{\sqrt{x^2 + a^2}} dx = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}) + C$$

$$36. \int \frac{x^2}{\sqrt{(x^2 + a^2)^3}} dx = -\frac{x}{\sqrt{x^2 + a^2}} + \ln(x + \sqrt{x^2 + a^2}) + C$$

$$37. \int \frac{dx}{x\sqrt{x^2 + a^2}} = \frac{1}{a} \ln \frac{\sqrt{x^2 + a^2} - a}{|x|} + C$$

$$38. \int \frac{dx}{x^2 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{a^2 x} + C$$

$$39. \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}) + C$$

$$40. \int \sqrt{(x^2 + a^2)^3} dx = \frac{x}{8} (2x^2 + 5a^2) \sqrt{x^2 + a^2} + \frac{3}{8} a^4 \ln(x + \sqrt{x^2 + a^2}) + C$$

$$41. \int x \sqrt{x^2 + a^2} dx = \frac{1}{3} \sqrt{(x^2 + a^2)^3} + C$$

$$42. \int x^2 \sqrt{x^2 + a^2} dx = \frac{x}{8} (2x^2 + a^2) \sqrt{x^2 + a^2} - \frac{a^4}{8} \ln(x + \sqrt{x^2 + a^2}) + C$$

$$43. \int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} + a \ln \frac{\sqrt{x^2 + a^2} - a}{|x|} + C$$

$$44. \int \frac{\sqrt{x^2 + a^2}}{x^2} dx = -\frac{\sqrt{x^2 + a^2}}{x} + \ln(x + \sqrt{x^2 + a^2}) + C$$

7 含有  $\sqrt{x^2 - a^2}$  ( $a > 0$ ) 的积分

$$45. \int \frac{dx}{\sqrt{x^2 - a^2}} = \frac{x}{|x|} \operatorname{arch} \frac{|x|}{a} + C_1 = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$46. \int \frac{dx}{\sqrt{(x^2 - a^2)^3}} = -\frac{x}{a^2 \sqrt{x^2 - a^2}} + C$$

$$47. \int \frac{x}{\sqrt{x^2 - a^2}} dx = \sqrt{x^2 - a^2} + C$$

$$48. \int \frac{x}{\sqrt{(x^2 - a^2)^3}} dx = -\frac{1}{\sqrt{x^2 - a^2}} + C$$

$$49. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$50. \int \frac{x^2}{\sqrt{(x^2 - a^2)^3}} dx = -\frac{x}{\sqrt{x^2 - a^2}} + \ln|x + \sqrt{x^2 - a^2}| + C$$

$$51. \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \arccos \frac{a}{|x|} + C$$

$$52. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$

$$53. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$54. \int \sqrt{(x^2 - a^2)^3} dx = \frac{x}{8} (2x^2 - 5a^2) \sqrt{x^2 - a^2} + \frac{3}{8} a^4 \ln|x + \sqrt{x^2 - a^2}| + C$$

$$55. \int x \sqrt{x^2 - a^2} dx = \frac{1}{3} \sqrt{(x^2 - a^2)^3} + C$$

$$56. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$57. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - \arccos \frac{a}{|x|} + C$$

$$58. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = -\frac{\sqrt{x^2 - a^2}}{x} + \ln |x + \sqrt{x^2 - a^2}| + C$$

## 8 含有 $\sqrt{a^2 - x^2}$ ( $a > 0$ ) 的积分

$$59. \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$60. \int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$$

$$61. \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + C$$

$$62. \int \frac{x}{\sqrt{(a^2 - x^2)^3}} dx = -\frac{1}{\sqrt{a^2 - x^2}} + C$$

$$63. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$64. \int \frac{x^2}{\sqrt{(a^2 - x^2)^3}} dx = \frac{x}{\sqrt{a^2 - x^2}} - \arcsin \frac{x}{a} + C$$

$$65. \int \frac{dx}{x \sqrt{a^2 - x^2}} = \frac{1}{a} \ln \frac{a - \sqrt{a^2 - x^2}}{|x|} + C$$

$$66. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{a^2 - x^2}{a^2 x} + C$$

$$67. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$68. \int \sqrt{(a^2 - x^2)^3} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3}{8} a^4 \arcsin \frac{x}{a} + C$$

$$69. \int x \sqrt{a^2 - x^2} dx = -\frac{1}{3} \sqrt{(a^2 - x^2)^3} + C$$

$$70. \int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} + C$$

$$71. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} + a \ln \frac{a - \sqrt{a^2 - x^2}}{|x|} + C$$

$$72. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{a} + C$$

## 9 含有 $\sqrt{\pm ax^2 + bx + c}$ ( $a > 0$ ) 的积分

$$73. \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{a}} \ln |2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}| + C$$

$$74. \int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} \\ + \frac{4ac - b^2}{8\sqrt{a^3}} \ln |2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}| + C$$

$$75. \int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} \\ - \frac{b}{2\sqrt{a^3}} \ln |2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}| + C$$

$$76. \int \frac{dx}{\sqrt{c + bx - ax^2}} = \frac{1}{\sqrt{a}} \arcsin \frac{2ax - b}{\sqrt{b^2 + 4ac}} + C$$

$$77. \int \sqrt{c + bx - ax^2} dx = \frac{2ax - b}{4a} \sqrt{c + bx - ax^2} + \frac{b^2 + 4ac}{8\sqrt{a^3}} \arcsin \frac{2ax - b}{\sqrt{b^2 + 4ac}} + C$$

$$78. \int \frac{x}{\sqrt{c + bx - ax^2}} dx = -\frac{1}{a} \sqrt{c + bx - ax^2} + \frac{b}{2\sqrt{a^3}} \arcsin \frac{2ax - b}{\sqrt{b^2 + 4ac}} + C$$



## 10 含有 $\sqrt{\pm \frac{x-a}{x+a}}$ 或者 $\sqrt{(x-a)(b-x)}$ 的积分

$$79. \int \sqrt{\frac{x-a}{x-b}} dx = (x-b) \sqrt{\frac{x-a}{x-b}} + (b-a) \ln(\sqrt{|x-a|} + \sqrt{|x-b|}) + C$$

$$80. \int \sqrt{\frac{x-a}{x-b}} dx = (x-b) \sqrt{\frac{x-a}{x-b}} + (b-a) \arcsin \sqrt{\frac{x-a}{b-a}} + C$$

$$81. \int \frac{dx}{\sqrt{(x-a)(x-b)}} = 2 \arcsin \sqrt{\frac{x-a}{b-a}} + C \quad (a < b)$$

$$82. \int \sqrt{(x-a)(b-x)} dx = \frac{2x-a-b}{4} \sqrt{(x-a)(b-x)} \\ + \frac{(b-a)^2}{4} \arcsin \sqrt{\frac{x-a}{b-a}} + C \quad (a < b)$$

## 11 含有三角函数函数的积分

$$83. \int \sin x dx = -\cos x + C$$

$$84. \int \cos x dx = \sin x + C$$

$$85. \int \tan x dx = \ln |\cos x| + C$$

$$86. \int \operatorname{ctg} x dx = \ln |\sin x| + C$$

$$87. \int \sec x dx = \ln \left| \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right| + C = \ln |\sec x + \tan x| + C$$

$$88. \int \csc x dx = \ln \left| \tan \frac{x}{2} \right| + C = \ln |\csc x - \operatorname{ctg} x| + C$$

$$89. \int \sec^2 x dx = \tan x + C$$

$$90. \int \csc^2 x dx = -\cot x + C$$

$$91. \int \sec x \tan x dx = \sec x + C$$

$$92. \int \csc x dx \cot x dx = -\csc x + C$$

$$93. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C$$

$$94. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C$$

$$95. \int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$96. \int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$97. \int \frac{dx}{\sin^n x} = -\frac{1}{n-1} \cdot \frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}$$

$$98. \int \frac{dx}{\cos^n x} = \frac{1}{n-1} \cdot \frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}$$

$$99. \int \cos^m \sin^n x dx = \frac{1}{m+n} \cos^{m-1} x \sin^{n+1} x + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x dx$$

$$= -\frac{1}{m+1} \cos^{m+1} x \sin^{n-1} x + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x dx$$

$$100. \int \sin ax \cos bx dx = -\frac{1}{2(a+b)} \cos(a+b)x - \frac{1}{2(a-b)} \cos(a-b)x + C$$

$$101. \int \sin ax \sin bxdx = -\frac{1}{2(a+b)} \sin(a+b)x + \frac{1}{2(a-b)} \sin(a-b)x + C$$

$$102. \int \cos ax \cos bxdx = \frac{1}{2(a+b)} \sin(a+b)x + \frac{1}{2(a-b)} \sin(a-b)x + C$$

$$103. \int \frac{dx}{a+b \sin x} = \frac{2}{\sqrt{a^2-b^2}} \arctan \frac{\arctan \frac{x}{2} + b}{\sqrt{a^2-b^2}} + C \quad (a^2 > b^2)$$

$$104. \int \frac{dx}{a+b \sin x} = \frac{1}{\sqrt{b^2-a^2}} \ln \left| \frac{\arctan \frac{x}{2} + b - \sqrt{b^2-a^2}}{\arctan \frac{x}{2} + b + \sqrt{b^2-a^2}} \right| + C \quad (a^2 < b^2)$$

$$105. \int \frac{dx}{a+b \cos x} = \frac{2}{a+b} \sqrt{\frac{a+b}{a-b}} \arctan \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) + C \quad (a^2 > b^2)$$

$$106. \int \frac{dx}{a+b \cos x} = \frac{1}{a+b} \sqrt{\frac{a+b}{b-a}} \ln \left| \frac{\tan \frac{x}{2} + \sqrt{\frac{a+b}{b-a}}}{\tan \frac{x}{2} - \sqrt{\frac{a+b}{b-a}}} \right| + C \quad (a^2 < b^2)$$

$$107. \int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{1}{ab} \arctan \left( \frac{b}{a} \tan x \right) + C$$

$$108. \int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} = \frac{1}{2ab} \ln \left| \frac{b \tan x + a}{b \tan x - a} \right| + C$$

$$109. \int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{1}{a} x \cos ax + C$$

$$110. \int x^2 \sin ax dx = -\frac{1}{a} x^2 \cos ax + \frac{2}{a^2} x \sin ax + \frac{2}{a^3} \cos ax + C$$

$$111. \int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{1}{a} x \sin ax + C$$

$$112. \int x^2 \cos ax dx = \frac{1}{a} x^2 \sin ax + \frac{2}{a^2} x \cos ax - \frac{2}{a^3} \sin ax + C$$

## 12 含有反三角函数的积分 (其中 $a > 0$ )

$$113. \int \arcsin \frac{x}{a} dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2} + C$$

$$114. \int x \arcsin \frac{x}{a} dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \arcsin \frac{x}{a} + \frac{x}{4} \sqrt{a^2 - x^2} + C$$

$$115. \int x^2 \arcsin \frac{x}{a} dx = \frac{x^3}{3} \arcsin \frac{x}{a} + \frac{1}{9} (x^2 + 2a^2) \sqrt{a^2 - x^2} + C$$

$$116. \int \arccos \frac{x}{a} dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} + C$$

$$117. \int x \arccos \frac{x}{a} dx = \left( \frac{x^2}{2} - \frac{a^2}{4} \right) \arccos \frac{x}{a} - \frac{x}{4} \sqrt{a^2 - x^2} + C$$

$$118. \int x^2 \arccos \frac{x}{a} dx = \frac{x^3}{a} \arccos \frac{x}{a} - \frac{1}{9} (x^2 + 2a^2) \sqrt{a^2 - x^2} + C$$

$$119. \int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2) + C$$

$$120. \int x \arctan \frac{x}{a} dx = \frac{1}{2} (a^2 + x^2) \arctan \frac{x}{a} - \frac{a}{2} x + C$$

$$121. \int x^2 \arctan \frac{x}{a} dx = \frac{x^3}{3} \arctan \frac{x}{a} - \frac{a}{6} x^2 + \frac{a^3}{6} \ln(a^2 + x^2) + C$$

## 13 含有指数函数的积分

$$122. \int a^x dx = \frac{1}{\ln a} a^x + C$$

$$123. \int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$124. \int x e^{ax} dx = \frac{1}{a^2} (ax - 1) e^{ax} + c$$

$$125. \int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$126. \int x a^x dx = \frac{x}{\ln a} a^x - \frac{1}{(\ln a)^2} a^x + C$$

$$127. \int x^n a^x dx = \frac{1}{\ln a} x^n a^x - \frac{n}{\ln a} \int x^{n-1} a^x dx$$

$$128. \int e^{ax} \sin bxdx = \frac{1}{a^2 + b^2} e^{ax} (a \sin bx - b \cos bx) + C$$

$$129. \int e^{ax} \cos bxdx = \frac{1}{a^2 + b^2} e^{ax} (b \sin bx + a \cos bx) + C$$

$$130. \int e^{ax} \sin^n bxdx = \frac{1}{a^2 + b^2 n^2} e^{ax} \sin^{n-1} bx (a \sin bx - nb \cos bx) \\ + \frac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \sin^{n-2} bxdx$$

$$131. \int e^{ax} \cos^n bxdx = \frac{1}{a^2 + b^2 n^2} e^{ax} \cos^{n-1} bx (a \cos bx + nb \sin bx) \\ + \frac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \cos^{n-2} bxdx$$

## 14 含有对数函数的积分

$$132. \int \ln x dx = x \ln x - x + C$$

$$133. \int \frac{dx}{x \ln x} = \ln |\ln x| + C$$

$$134. \int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \left( \ln x - \frac{1}{n+1} \right) + C$$

$$135. \int (\ln x)^n dx = x (\ln x)^n - n \int (\ln x)^{n-1} dx$$

$$136. \int x^m (\ln x)^n dx = \frac{1}{m+1} x^{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx$$

## 15 含有双曲函数的积分

$$137. \int \sinh x dx = \cosh x + C$$

$$138. \int \cosh x dx = \sinh x + C$$

$$139. \int \tanh x dx = \ln \cosh x + C$$

$$140. \int \sinh^2 x dx = -\frac{x}{2} + \frac{1}{4} \sinh 2x + C$$

$$141. \int \cosh^2 x dx = \frac{x}{2} + \frac{1}{4} \sinh 2x + C$$

## 16 定积分

$$142. \int_{-\pi}^{\pi} \cos nx dx = \int_{-\pi}^{\pi} \sin nx dx = 0$$

$$143. \int_{-\pi}^{\pi} \cos mx \sin nx dx = 0$$

$$144. \int_{-\pi}^{\pi} \cos mx \cos nx dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$145. \int_{-\pi}^{\pi} \sin mx \sin nx dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$146. \int_0^{\pi} \sin mx \sin nx dx = \int_0^{\pi} \cos mx \cos nx dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = n \end{cases}$$

$$147. I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx$$

$$I_n = \frac{n-1}{n} I_{n-2}$$

$$\begin{cases} I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{4}{5} \cdot \frac{2}{3} & (n \text{ 为大于 } 1 \text{ 的正奇数}), I_1 = 1 \\ I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} & (n \text{ 为正偶数}), I_0 = \frac{\pi}{2} \end{cases}$$